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Working Group Summary: Pion-Nucleon Coupling Constant

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Abstract

A brief introduction to different determinations of the πNN coupling constant is given, and some comments on the topics discussed in the working group are made.

INTRODUCTION

Since the birth of the Yukawa theory of the nuclear force in 1935 it was a challenge for the physics community to determine the coupling strength of the Yukawa meson to the nucleon. In 1947 the meson – pion – was finally discovered in cosmic ray emulsion experiments[1] and more systematic work to determine the πNN coupling constant could start. Conventionally[2] the pseudoscalar strength is denoted by g and the pseudovector coupling constant by f such that

$$f^2 = \left(\frac{M_\pi}{2m_p}\right)^2 \frac{g^2}{4\pi}, \tag{1}$$

where M_π is the charged pion mass and m_p is the proton mass. Other conventions concerning the nucleon mass and the factor 4π appear in the literature[3]. Reasonable estimates for the coupling strength were obtained even before the discovery of the pion and without detailed knowledge of the meson mass, e.g., Bethe was able to get an estimate $f^2 = 0.077$ - 0.080 already in 1940[4] on the basis of deuteron properties. The results of various determinations until 1980 are shown in Fig. 1. In the same figure very different techniques to determine f^2 are summarized. In the previous *MENU* symposium de Swart gave a review on the topic[3] and many of the references used in Fig. 1 can be found there. The values of

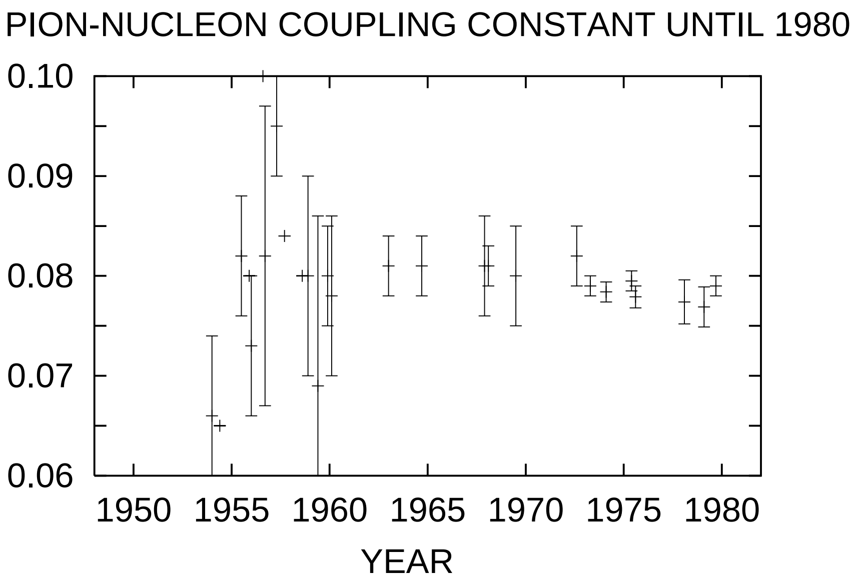

Figure. 1. The values of the pion-nucleon coupling constant f^2 before 1980.

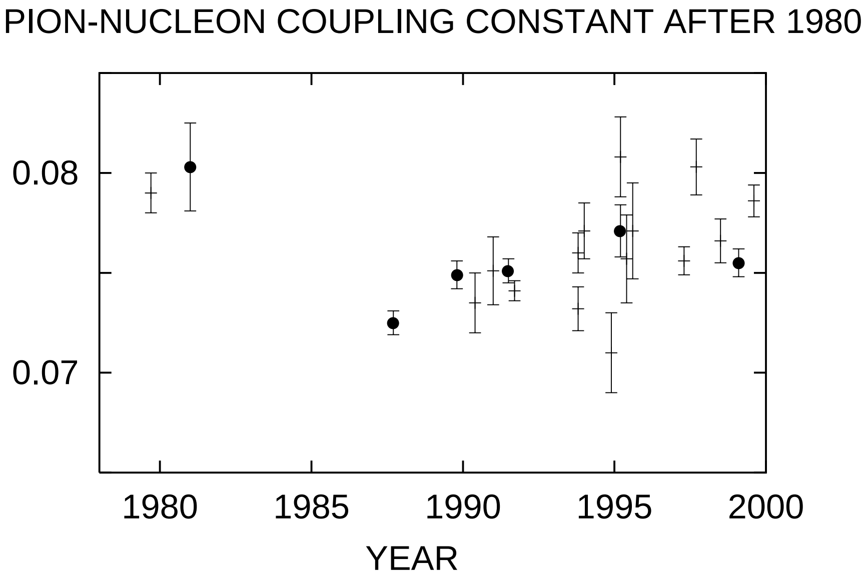

Figure. 2. The values of the pion-nucleon coupling constant f^2 after 1980 until the present. Neutral pion couplings are denoted by the solid dots, the remaining points refer to charged pion couplings or charge independent determinations.

f^2 stabilized for a long time[5–7] and only in the 90's has the discussion of the value of the pion-nucleon coupling constant started again. In[5–7] fixed-t dispersion relations for πN were used. In the determinations displayed in Fig. 1 most of the data date back to the era before the meson factories, LAMPF, SIN and TRIUMF, which, in addition to performing experiments with pions, had programmes to study the NN interaction. In several analyses shown in Fig. 2 NN scattering data have been used to extract the πN coupling strength, i.e. one of the standard methods until the 60's has been adopted again in more refined form. In this activity the Nijmegen group has played an important role[3]. Of course, in Fig. 2 many results from the meson factory πN experiments are included as well in the data bases used to determine f^2.

The central issue in the discussion in the working group has been the scatter of the results of the determinations as shown in Fig. 2. The main questions involve the model dependence of different techniques, the effect of different pieces of data (partly conflicting), the error estimates, the electromagnetic corrections and other isospin violating effects. In the working group contributions were presented by Loiseau[8], Höhler[9] and Pavan[10], and brief commentaries by W.R. Gibbs, M. Birse and D.V. Bugg.

PROBLEMS IN EXTRACTING f^2

The particular issues raised in the discussion include:

- Electromagnetic corrections:
  - πN vs. NN; the different treatment of electromagnetic corrections for these two scattering processes gives a possibility to check the uncertainty due to these effects
  - corrections from Tromborg et al.[11] vs. Oades et al.[12]; the dispersion approach and potential model lead to differences which need checking
  - corrections at high energy; these need to be checked, Tromborg et al. calculated corrections only up to 655 MeV/c
- Lack of transparency of the analyses; the analyses contain large data bases and it is hard to clarify which pieces of information are the crucial ones in determining f^2

- The normalization of the np data is a problem in the (p,n) data analyses
- The determination of the s-wave isoscalar scattering length, a^+_{0+}, from the $\pi^- d$ level shift measurement suffers from some model dependence due to electromagnetic and absorption corrections
- Effective theory is not at present suitable for fixing the coupling constant. The problems relate mostly to the convergence of the chiral expansion or to the additional low-energy constants which are not known accurately enough. However, there might be a chance in a precise measurement of the induced pseudoscalar coupling constant, g_P, which would make an accurate determination of the pion-nucleon coupling constant possible[13]
- There is need for a new fixed-t analysis of πN scattering data which extends beyond the present limit of the VPI analysis, 2.1 GeV
- There is need for a new analysis of the forward dispersion relations of the NN system. The amount of NN data has increased considerably since the previous analysis thanks to the meson factories and SATURNE.

The GMO Sum Rule

The Goldberger-Miyazawa-Oehme sum rule (GMO)[14] provides a simple means to estimate the pion-nucleon coupling constant directly from measurable quantities, the πN isovector s-wave scattering length and total cross sections from the threshold to the highest energies. The method still has uncertainties, and will probably never be able to compete with other methods in precision, but the advantage is the possibility to relate the uncertainty in f^2 directly to the experimental errors.

The GMO sum rule is the result of the forward dispersion relation for the $D^-(= A^- + \nu B^-)$ amplitude taken at the physical threshold (the total laboratory energy $\omega = M_\pi$)

$$D^-(M_\pi) = \frac{8\pi f^2}{M_\pi[1-(M_\pi^2/4m_p^2)]} + 4\pi M_\pi J^- = 4\pi(1+x)a^-_{0+}\,, \tag{2}$$

where

$$J^- = \frac{1}{2\pi^2}\int_0^\infty \frac{\sigma^-(k)}{\omega}\,dk \tag{3}$$

and $x = M_\pi/m_p$. The pion-nucleon coupling constant can now be extracted and the result is

$$\begin{aligned} f^2 &= \frac{1}{2}[1-(\frac{x}{2})^2][(1+x)M_\pi a^-_{0+} - M_\pi^2 J^-] \\ &= 0.5712(M_\pi\, a^-_{0+}) - 0.02488(J^-/\mathrm{mb}). \end{aligned} \tag{4}$$

The isovector s-wave scattering length, a^-_{0+}, is accessible through experiment[19]. For the integral J^- several evaluations are displayed in Table 1. As can be seen from Fig. 3 there is potential sensitivity to details of the electromagnetic corrections especially around the Δ-resonance region near 0.3 GeV/c as well as to the treatment of the Δ^{++}, Δ^0 splitting. Making use of the isospin symmetry gives for the scattering length $a^-_{0+} = 0.0962 \pm 0.0071\, M_\pi^{-1}$[19] and taking Koch's value for J^- gives an estimate for the lower limit of the coupling constant f^2 with the result 0.0765. With more conservative errors for J^- the figure 0.0762 is obtained. With the remaining uncertainties in the treatment of various corrections this limit is not in real conflict with the results from other analyses.

Table 1. Values for the J^- integral.

Ref.	J^- (mb)
KH ('83)[2]	-1.058
Koch ('85)[15]	-1.077 $\pm$ 0.047
VPI ('92)[16]	-1.072
Gibbs ('98)[17]	-1.051
ELT ('99)[18]	-1.083 $\pm$ 0.025

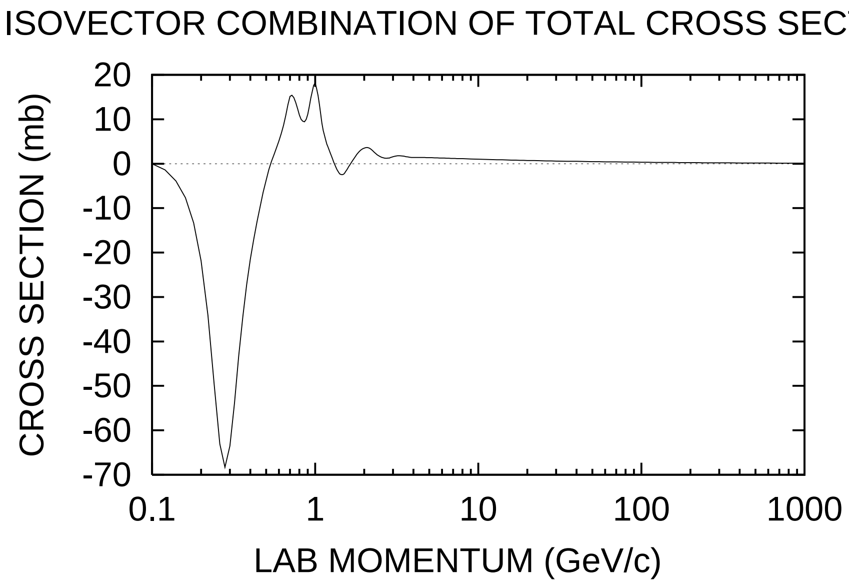

Figure. 3. The isovector combination, $\sigma^- = \frac{1}{2}(\sigma_{\pi^- p} - \sigma_{\pi^+ p})$, of the $\pi^- p$ and $\pi^+ p$ total cross sections[2]. Experimental data extend up to 350 GeV/c.

CONCLUDING REMARKS

The precision of the pion-nucleon experiments has now reached the level where a more careful treatment of the corrections, in particular of electromagnetic origin or due to the u- and d-quark mass difference, is necessary. These theoretical challenges have not yet been met, eventhough the theoretical tool, chiral perturbation theory, has now the capability to answer these questions. Work along these lines is in progress. In the lattice[20] and the QCD sum rule[21] frontiers the present accuracy for g is 20-30 % and it will take a while before this improves significantly.

Table 2 summarizes some recent values for f^2 displayed in Fig. 2. The table demonstrates the current trend, the favoured value for f^2 is slightly smaller than the standard one of Koch and Pietarinen[7]. However, there remains still quite a number of problems which need attention.

The Goldberger-Treiman discrepancy

$$\Delta_{\pi N} = 1 - \frac{m_p \, g_A}{F_\pi \, g}, \tag{5}$$

where F_π and g_A are the pion and neutron decay constants respectively, would be reduced from 4 % to 2 %, if f^2 changes from 0.079 to 0.076. In a recent SU(3) analysis[27] preference for a smaller Goldberger-Treiman discrepancy was found.

In the analysis of np scattering data at backward directions somewhat higher value for the coupling constant has been obtained[28], $f^2 = 0.0803 \pm 0.0014$. Discussion on

Table 2. Values for the pion-nucleon coupling constant f^2 from recent determinations.

Ref.	f^2	Method
KH ('80)[7]	0.079 ± 0.001	πN fixed-t
BM ('95)[22]	0.0757 ± 0.0022	NN data
Gibbs ('98)[17]	0.0756 ± 0.0007	GMO
Machner ('98)[23]	0.0760 ± 0.0011	symmetries
Matsinos ('98)[24]	0.0766 ± 0.0011	model fit
Nijmegen ('99)[25]	0.0756 ± 0.0004	pp PWA
ELT ('99)[18]	0.0786 ± 0.0008	GMO + $\pi^- d$
VPI ('99)[10,26]	0.0760 ± 0.0004	πN fixed-t

the problems in this field continues[29,30]. The spin transfer coefficients in pp scattering are also of interest, the preliminary indications are towards slightly smaller value for the coupling[31]. Machleidt has recently discussed[32] some additional problems with the deuteron properties and low-energy NN analyzing powers which indicate that no coherent picture is yet emerging.

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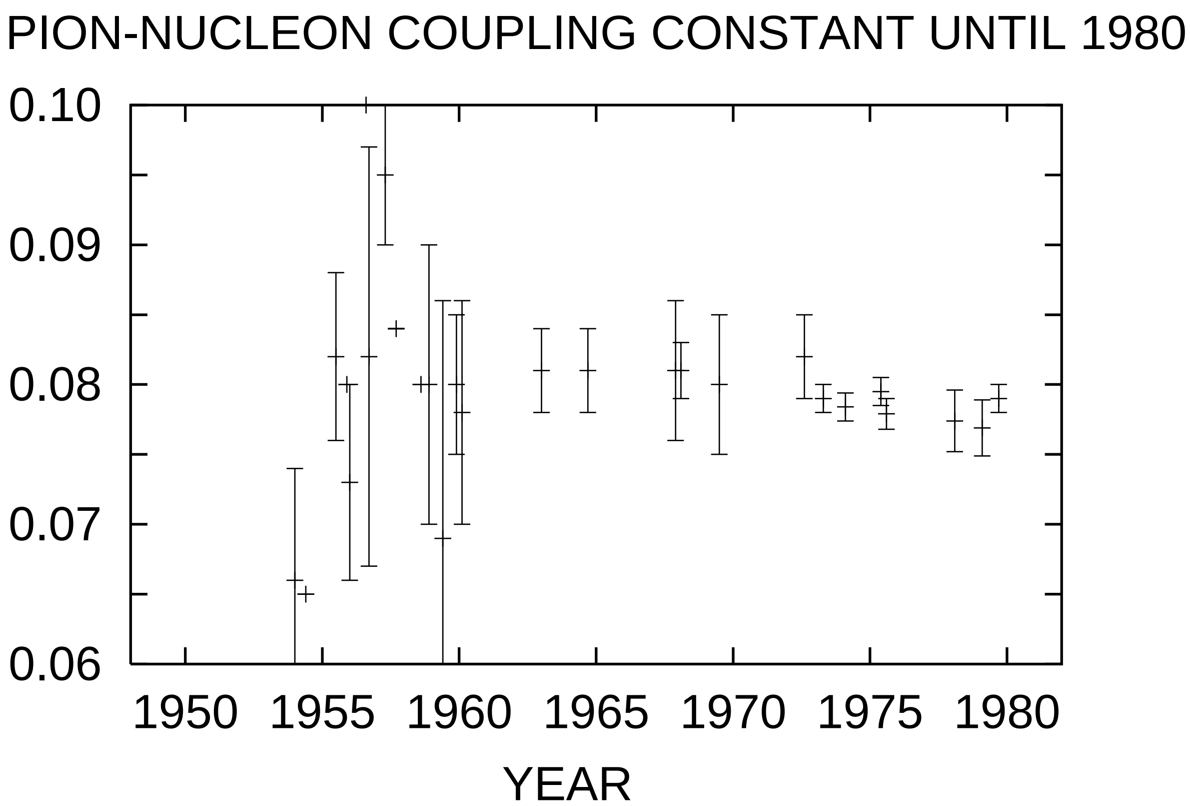
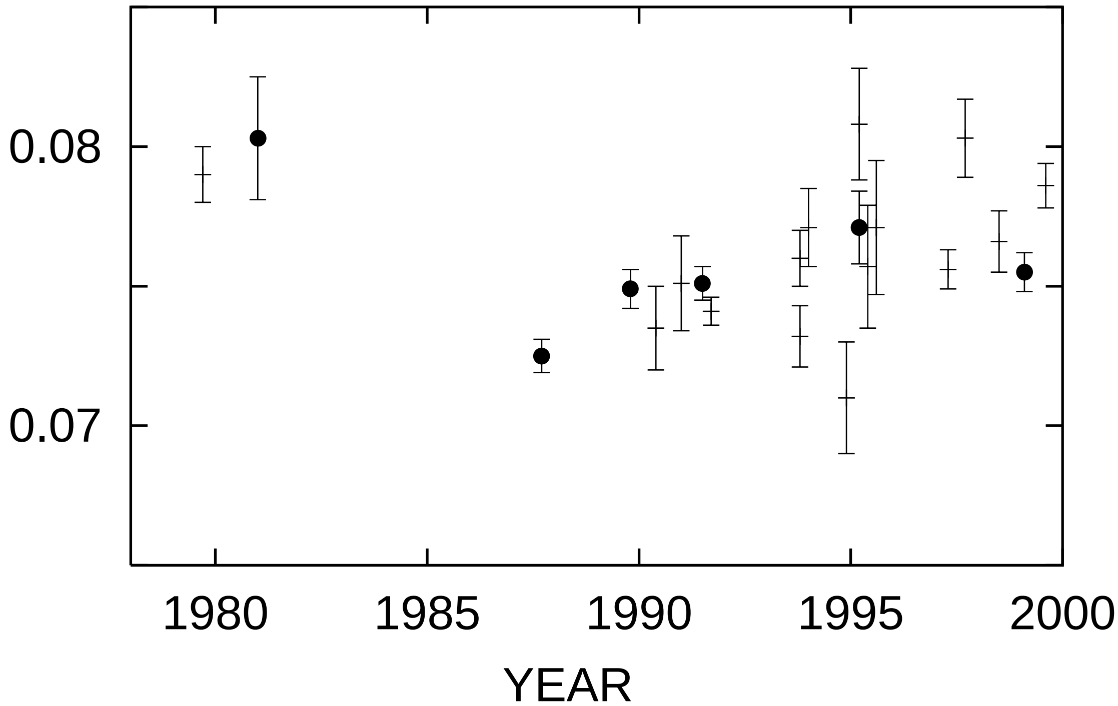
REFERENCES

1. C.M.G Lattes, H. Muirhead, G.P.S. Occhialini, and C.F. Powell, "Processes involving charged mesons," Nature **159**, 694 (1947).
2. G. Höhler, *Landolt-Börnstein*, Vol. 9 b2, ed. H. Schopper (Springer, Berlin, 1983).
3. J.J. de Swart, M.C.M. Rentmeester, and R.G.E. Timmermans, "The status of the pion-nucleon coupling constant," in *Proceedings of MENU97* (TRIUMF, Vancouver, 1997) pp. 96–107; nucl-th/9802084.
4. H.A. Bethe, "The meson theory of nuclear forces: Part II. Theory of the deuteron," Phys. Rev. **57**, 390 (1940).
5. J. Hamilton, and W.S Woolcock, "Determination of pion-nucleon parameters and phase shifts by dispersion relations," Rev. Mod. Phys. **35**, 737 (1963).
6. D.V. Bugg, A.A. Carter, and J.R. Carter, "New values of pion-nucleon scattering lengths and f^2," Phys. Lett. **44B**, 278 (1973).
7. R. Koch, and E. Pietarinen, "Low-energy πN partial wave analysis," Nucl. Phys. **A336**, 331 (1980).
8. B. Loiseau, "How precisely can we determine the πNN coupling constant from isovector GMO sum rule?" *these proceedings.*
9. G. Höhler, "Determinations of the πNN coupling constant," *these proceedings.*
10. M. Pavan, "Determination of the πNN coupling constant in the VPI/GW $\pi N \to \pi N$ partial-wave and dispersion relation analysis," *these proceedings.*
11. B. Tromborg, S. Waldenstrøm, and I. Øverbø, "Electromagnetic corrections to πN scattering," Phys. Rev. **D15**, 725 (1977).
12. G.C. Oades, *these proceedings.*
13. V. Bernard, N. Kaiser, U.-G. Meißner, "QCD accurately predicts the induced pseudoscalar coupling constant," Phys. Rev. **D50**, 6899 (1994).
14. M.L. Goldberger, H. Miyazawa, and R. Oehme, "Application of dispersion relations to pion-nucleon scattering," Phys. Rev. **99**, 986 (1955).
15. R. Koch, "Inconsistencies in low-energy pion-nucleon scattering," Karlsruhe preprint TKP 85-5 (1985).
16. R.L. Workman, R.A. Arndt, and M.M. Pavan, "On the Goldberger-Miyazawa-Oehme sum rule," Phys. Rev. Lett. **68**, 1653 (1992); (E) ibid. **68**, 2712 (1992).
17. W.R. Gibbs, Li Ai, and W.B. Kaufmann, "Low-energy pion-nucleon scattering," Phys. Rev. **C57**, 784 (1998).

18. T.E.O. Ericson, B. Loiseau, and A.W. Thomas, "Precision determination of the πN scattering lengths and the charged πNN coupling constant," hep-ph/9907433.
19. D. Sigg et al., "The strong interaction shift and width of the ground state of pionic hydrogen," Nucl. Phys. **A609**, 269 (1996); (E) ibid. **A617**, 526 (1997).
20. K.F. Liu et al., "Valence QCD: Connecting QCD to the quark model," Phys. Rev. **D59**, 112001 (1999).
21. M.C. Birse, and B. Krippa, "Determination of pion-baryon coupling constants from QCD sum rules," Phys. Rev. **C54**, 3240 (1996).
22. D.V. Bugg, and R. Machleidt, "πNN coupling constants from NN elastic scattering data between 210 and 800 MeV," Phys. Rev. **C52**, 1203 (1995).
23. H. Machner, "Symmetries in low energy pion physics and the πNN coupling constant," Acta Phys. Pol. **B29**, 3081 (1998).
24. E. Matsinos, "The low-energy constants of the πN system," hep-ph/9807395.
25. M.C.M Rentmeester, R.G.E. Timmermans, J.L. Friar, and J.J. de Swart, "Chiral two-pion exchange and proton-proton partial-wave analysis," Phys. Rev. Lett. **82**, 4992 (1999).
26. M.M. Pavan, R.A. Arndt, I.I. Strakovsky, and R.L Workman, "Determination of the πNN coupling constant in the VPI/GW $\pi\mathrm{N} \to \pi\mathrm{N}$ partial-wave and dispersion relation analysis," nucl-th/9910040.
27. J.L. Goity, R. Lewis, M. Schvellinger, and L. Zhang, "The Goldberger-Treiman discrepancy in SU(3)," Jefferson Lab. preprint JLAB-THY-98-51, hep-ph/9901374.
28. J. Rahm et al., "*np* scattering measurements at 162 MeV and the πNN coupling constant," Phys. Rev. **C57**, 1077 (1998).
29. M.C.M. Rentmeester, R.A.M. Klomp, and J.J. de Swart, "Comment on πNN coupling from high precision *np* charge exchange at 162 MeV," Phys. Rev. Lett. **81**, 5253 (1998).
30. T.E.O. Ericson et al., Phys. Rev. Lett. **81**, 5254 (1998).
31. S.W. Wissink, "Spin transfer coefficients for *pp* elastic scattering at 200 MeV: Implications for the πNN coupling constant," Nucl. Phys. **A631**, 411c (1998).
32. R. Machleidt, "How sensitive are various NN observables to changes in the πNN coupling constant?," nucl-th/9909036.


PION-NUCLEON COUPLING CONSTANT UNTIL 1980
0.10
0.09
0.08
0.07
0.06
1950
1955
1960
1965
1970
1975
1980
YEAR

PION-NUCLEON COUPLING CONSTANT AFTER 1980



ISOVECTOR COMBINATION OF TOTAL CROSS SECTIONS